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Planning and Design of Flex-Route Transit Services

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A theoretical investigation is presented of various issues involved in the planning and design of flex-route transit services. An analytical model is proposed for an idealized operating environment with the objective of determining the optimal slack time that should be allocated to a flex-route segment. The optimization objective is defined to minimize total operator and user cost, which enables a systematic examination of complex interactions among the system parameters. An equation is derived for the relationship between the number of feasible deviations and various system parameters such as slack time, zone size, and dwell time. Subsequent analysis shows that the analytical model is elaborate enough to provide substantial insights into various issues that may arise in designing a flex-route service. A simulation analysis is conducted to validate some of the conclusions drawn from the analytical model and to further analyze the implications of stochastic variation in passenger demand.

Flex-route transit is a hybrid of conventional fixed-route transit and demand-responsive paratransit service. It assimilates conventional transit in that its main route covers a service corridor with a set of fixed stops and schedules. Flex-route transit is also demand responsive as its service vehicles are allowed to deviate from the main route to provide door-to-door or checkpoint-to-checkpoint service to users who either have trip ends located out of the service coverage of the main route or require accessible services such as paratransit. Thus, flex-route service is targeted to two groups of riders: one includes mainly the general public transit users who use the fixed stops and the other includes mostly the paratransit users who use the deviation service. By integrating the regularity of conventional fixed-schedule transit and the flexibility of demand-responsive–variable-route paratransit, flex-route transit has the potential to become a vital transit option, especially for low-density areas where demand for general public transit is too low to be efficiently serviced by conventional transit. The major economic potential of flex-route service is that it may reduce systemwide costs by covering a proportion of paratransit riders who would otherwise have to be accommodated by a dedicated but more costly paratransit service (*1, 2*).

Although conceptually simple and attractive, flex-route transit has proven to be much more difficult to plan, design, and operate than its counterpart modes—regular transit and paratransit. The reason is twofold. First, a flex-route service has to maintain a balance between the needs of the two groups of riders—the general public transit users and the paratransit users who usually have very different service requirements, expectations, and attitudes toward transit service. Second, operating a flex-route service requires a dedicated real-time decision support system responsible for registration, trip booking, scheduling, and dispatching. The real-time scheduling and dispatching tasks can become especially challenging when the system has to deal with a large volume of requests for deviation and highly varied demand at fixed stops. This research deals with the problem of planning and designing flex-route services, with the specific focus on issues such as where to set fixed stops, what zone size to use, and how much slack time to allocate.

The flex-route design problem has been addressed in several past studies involving empirical analysis and field experiments. Rosenbloom (*1*) provided a recent account of current practices on the flex-route service concept and acknowledged that the implementation of flex-route services is still limited to rural and small urban areas. The main research on flex-route transit was recently initiated by Durvasula et al. (*3*), who provided an extensive discussion on critical issues and challenges involved in flex-route operations and conducted an empirical study using data from North Virginia's Potomac and Rappahannock Transportation Commission. Welch et al. (*4*) described a methodology for determining whether or not part of a conventional service route should be modified for out-of-direction travel. To the author's knowledge, no research has been attempted to formulate the flex-route service-design problem in a systematic manner.

The objective is to develop an analytical optimization model that can be used to identify critical factors and relationships that need to be considered in flex-route design. First discussed is how a simplified analytical model can be established over an idealized operating environment. The analytical model is subsequently used to analyze various design issues in flex-route service. Finally, simulation experiments are conducted to test some of the conclusions drawn from the analytical model and analyze the effect of random variation in passenger demand on the quality of a flex-route service.

ANALYTICAL MODEL

This section introduces an analytical model for designing and analyzing flex-route service systems. The model is derived on the basis of an idealized operating environment with the following settings (see Figure 1 and Table 1):

- Both flex-route and paratransit services are provided by the same operator. All paratransit trips need to be accommodated either by the flex-route service or by a specialized paratransit service.
- The flex-route service is to be operated within a rectangular service area of length l and width $2w$. The service area is covered by a uniformly distributed grid road network with a link travel speed of v.
- The main route of the flex-route service is located at the middle of the zone and includes one route segment (or one service zone) between two fixed stops: Stops A and B. The service vehicle departs from Stops A and B at prescheduled departure times. The difference between the departure times at Stops B and A is the scheduled running time, denoted as T, which is also defined as the analysis period

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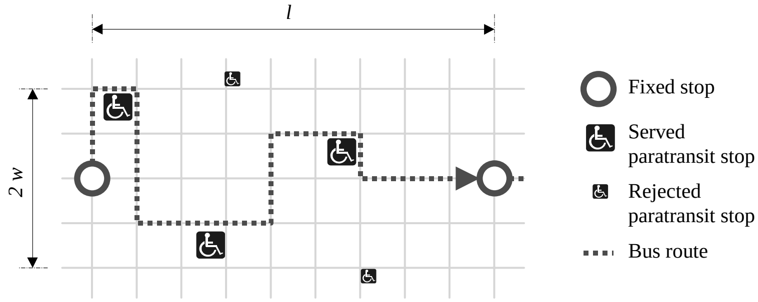

FIGURE 1 Flex-route service in an idealized network.

in this study. To accommodate possible deviations, the scheduled running time (T) must be greater than the direct running time between the fixed stops (T_0). The difference between them is called slack time, denoted by Δ ($T = T_0 + \Delta$), which represents the main decision variable in designing flex-route service.

- There are N_p paratransit stops that are expected during the analysis period (T, the scheduled time for a flex-route vehicle trip from Stops A to B). These deviated stops are to be serviced during the service headway. The stops are uniformly distributed over the service zone. There are N_t general public transit riders traveling from A to B for each flex-route trip. Both transit and paratransit demands are perfectly inelastic, that is, they are not affected by service quality.

With this assumed operating environment, the service-design problem is defined as to determine the optimal amount of slack time that should be allocated to this service route. The optimization objective is assumed to minimize the total net cost that would be incurred to the operator as well as to the passengers, as discussed in the following section. It is important to point out that the analysis performed in this study is not concerned with the feasibility of the flex-route service; that is, the decision to run the flex-route service is assumed to be made before this analysis. The question to be addressed is simply how to best operate the service.

Operator Cost

Because of route deviation, flex-route service incurs additional costs to the service operator. The total marginal operating cost depends on increases in mileage and service hours due to the route deviation service, and can be assumed to be proportional to the additional slack time (Δ) built into the schedule, that is,

TABLE 1 Variable Definitions

Symbol	Definition	Sample Value
l	Length of the service zone (km)	10.0
w	One-side width of the service zone representing zone size (km)	1.2
v	Average vehicle travel speed on road (km/h)	20.0
T_0	Direct running time between the fixed stops (h), $T_0 = l/v$	0.5
T	Scheduled running time between the fixed stops (h), $T = T_0 + \Delta$.	/
N_p	Total number of deviated stops requested per analysis period T	4
n_p	Total number of feasible deviations per analysis period	2
T_p	Expected total route length to visit n_p stops (h)	/
N_t	Number of transit riders from stop A to stop B per flex-route trip (or analysis period)	10
c_f	marginal hourly operating cost of flex-route service ($/hour)	12
c_p	the cost that would result from not servicing a deviated stop, or marginal benefit of servicing a deviated stop, or marginal operating cost of servicing a deviated stop using paratransit ($/stop)	8
c_t	Cost coefficient representing passengers' disutility toward increase in ride time due to deviation	10
C^o	Operator cost due to deviation from the main route for analysis period T ($)	/
B^o	Operator benefit due to covering deviated stops for each analysis period T ($)	/
C^u	User cost associated with transit riders due to increase in ride time for each analysis period T ($)	/
M	Vehicle capacity (seats)	20
Δ	Slack time - decision variable (hour)	/

NOTE: / = for corresponding variables, default values are either not available or not necessary.

$$C_f^o = c_f \cdot \Delta \tag{1}$$

where C_f^o is the total marginal cost for each analysis period T and c_f is the marginal hourly operating cost of the flex-route service. It should be emphasized that this marginal hourly operating cost should include only the increment in cost due to route deviation as compared with a service without route deviation (regular transit). It is different from the hourly operating cost of the flex-route service, which would include all costs such as vehicles, crews, and management. Therefore, the main contributor to the marginal cost should be the cost of gasoline.

Service Benefit

The benefit of operating a deviation service is that it will cover a certain number of trips that would otherwise not be served or have to be served by a more costly option, such as driving, regular transit (which means opening of new transit routes), or specialized paratransit. The benefit should therefore be equivalent to the costs that would result if the flex-route service was not provided. However, quantifying such costs is extremely difficult, if not impossible, especially in those cases in which the deviation service is open to the general public. For the general public, the tangible benefits of flex-route service could be increased mobility and reduced traffic congestion. But the question is how to measure the magnitude of those benefits. In this study, the analysis was limited to cases in which the deviation service was only for paratransit users and there was a dedicated paratransit service available to cover paratransit requests if not serviced by the flex-route service. With this assumption, we can assume the operator benefit, denoted by B_p^o, is a function of the number of paratransit trips that can be covered by the route deviation service and the marginal operating cost of paratransit service, that is,

$$B_p^o = n_p \cdot c_p \tag{2}$$

where c_p is the marginal operating cost of the paratransit service (\$/stop) and n_p is the number of feasible deviation that can be covered by route deviation during a flex-route trip. Note that the marginal operating cost of paratransit service can be readily obtained from paratransit service providers.

The number of deviated stops that can be covered (n_p) is limited by the amount of slack time allocated to the route segment. For operational efficiency, the amount of slack time allocated should be approximately equal to the difference between the expected route time (with n_p deviated stops) and the direct running time, that is,

$$\Delta = T_p - T_0 \tag{3}$$

where T_0 is the direct running time, $T_0 = l/v$; T_p is the expected time required to cover n_p deviated starting from Stop A and ending at Stop B. T_p depends on how the stops are routed or sequenced, which in turn depends on how the deviated stops are distributed over the zone. Under the idealized conditions assumed in this study, the route length can be approximated with the following simplified routing strategy: deviated stops are first ordered based on their horizontal distances from the starting stop (A) and then visited sequentially; once a stop is visited, the vehicle will return to the main route (center line) and then to the next stop via the shortest path (see Figure 1). With this routing strategy, the expected total time to visit n_p random stops is

$$\begin{aligned} T_p &= \frac{l + n_p \cdot w}{v} + n_p \cdot \tau \\ &= l/v + (w/v + \tau)\, n_p \\ &= T_0 + \delta\, n_p \end{aligned} \tag{4}$$

where

$\delta\ (= w/v + \tau)$ = expected time per deviation or the additional time needed to visit one additional deviated stop,
τ = average dwell time at a deviated stop, and
v = average vehicle travel speed.

Replace T_p in Equation 3 with Equation 4; the expected number of stops that can be serviced within a given amount of slack time can be obtained as follows:

$$n_p = \frac{T_p - T_0}{w/v + \tau} = \frac{\Delta}{\delta} \tag{5}$$

Equation 5 indicates that the number of feasible deviation is a linear function of the slack time. It should be noted that this relationship is approximate resulting from the continuous approximation over an idealized network. Further discussion on this result is included in the following simulation study.

With Equation 5, the expected operator benefit shown in Equation 2 can therefore be expressed as a function of slack time as follows:

$$B_p^o = \frac{\Delta}{\delta} \cdot c_p \tag{6}$$

In addition, the number of feasible deviations is also limited by the total number of available deviations requested ($n_p \le N_p$), and vehicle capacity ($n_p + N_t \le M$). With Equation 5, we can obtain the following constraints:

$$\Delta \le N_p \cdot \delta \tag{7}$$

$$\Delta \le (M - N_t) \cdot \delta \tag{8}$$

The number of paratransit riders that the flex-route system can accept is also limited by the vehicle seating capacity. To avoid this issue, we assume the vehicle size is not a limiting factor for providing the service; that is, the flex-route vehicles are large enough to handle all possible trips.

User Cost

Route deviation will however cause inconvenience to the transit riders because of increased ride time. The larger the deviation or slack time the higher the inconvenience will become. Such inconvenience could even lead to loss of transit riders when it exceeds a certain amount. It is therefore necessary to consider this consequence in designing a flex-route service. The inconvenience resulting from route deviation is modeled as a user cost which is assumed to be a function of the increase in transit rider travel time (Δ) as follows:

$$C^u = N_t \cdot c_t \cdot \Delta^r \tag{9}$$

where

N_t = average number of transit riders for each flex-route trip from Stop A to Stop B,
c_t = cost coefficient that can be calibrated on the basis of passenger attitude toward increased ride time, and
r = model parameter representing transit rider sensitivity to deviation time.

In this study, we assume $r = 1$, and consequently the corresponding cost coefficient c_t can be considered as the value of time of the transit riders.

A threshold is used to consider the maximum allowable deviation as follows:

$$\frac{\Delta}{T_0} \le \beta \qquad \text{or} \qquad \Delta \le T_0\beta \tag{10}$$

where β is the maximum allowable deviation ratio.

For the paratransit riders, quality of service provided by flex-route service and paratransit service are assumed similar and no user cost is therefore considered in this analysis.

Problem Formulation and Solution

The problem of identifying optimal slack time can now be formulated as a linear programming problem:

$$\begin{aligned} \min Z(\Delta) &= \text{Operator cost} + \text{User cost} - \text{Service benefit} \\ &= C_f^o + C^u - B_p^o \\ &= \left(N_t c_t - \frac{c_p - c_f\delta}{\delta}\right)\Delta \end{aligned} \tag{11}$$

subject to

$$0 \le \Delta \le \min\{N_p\delta, T_0\beta, (M - N_t)\delta\} \tag{12}$$

where Z is total marginal cost. This problem can be solved analytically, yielding the optimal slack time (Δ^*):

$$\Delta^* = \begin{cases} 0 & (c_p - c_f\delta)/\delta \le N_t c_t \\ \min\{N_p\delta, T_0\beta, (M - N_t)\delta\} & \text{otherwise} \end{cases} \tag{13}$$

where

$c_f\delta$ = unit operating cost of the flex-route service, or, the operating cost for the flex-route to service one additional deviated stop; as a result,
$(c_p - c_f\delta)/\delta$ = unit net operating benefit per deviation hour, and
$N_t c_t$ = the total cost of the transit riders per deviation hour.

Figure 2 shows an example relationship between total marginal cost and slack time when l = 5 km, w = 1 km , v = 20 km/h, c_f = \$12/hr , c_p = \$8/stop (or \$16/trip), c_t = \$6/hr (that is, transit riders' value of time is \$6/h, or they would "feel" a loss of \$1 for an increase of 10 min for deviation), N_t = 5 , N_p = 2, τ = 1 min, β = 40%, and M = 9 seats. The optimal slack time in this example is 6 min. A more detailed analysis is provided in the following section to show the application of this analytical result.

ANALYSIS WITH ANALYTICAL MODEL

The analytical model established previously, although simplistic, can provide some meaningful insights into various design issues involved in flex-route services. For example, in planning a flex-route service, it is often useful to be able to analyze how various service parameters such as service zone size and paratransit dwell time influence the number of deviations that can be accommodated by a flex-route service. The analytical model facilitates these types of investigations as described in the following section.

Optimal Slack Time

The optimal amount of slack time that should be provided to a route segment between two consecutive fixed stops is a function of many factors, including transit and paratransit demands, marginal operating costs of flex-route service and paratransit service, zone size, and paratransit dwell time. Indeed, on the basis of Equation 13, the deviation service would make sense only when the unit net operating

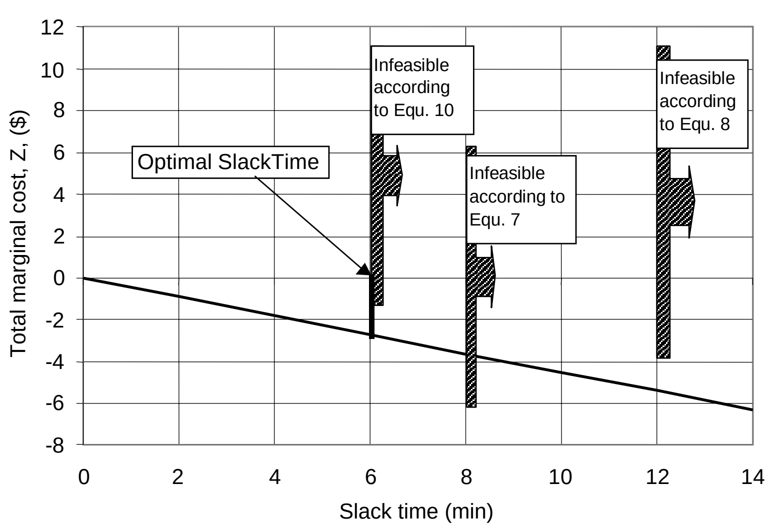

FIGURE 2 Optimal slack time.

benefit of the flex-route transit is greater than the collective value of time of the transit riders, that is,

$$\frac{(c_p - c_f \delta)}{\delta} > N_t c_t \tag{14}$$

An interesting observation of Equation 13 is that the economic viability of flex-route transit does not depend on paratransit demand, but the transit demand only. However, when deviation is in the beneficial region, the slack time should be as large as possible, when this is feasible [limited by maximum acceptable deviation for transit riders, $T_0\beta$, and capacity, $(M - N_t)\delta$] and meaningful (limited by demand, $N_p\delta$). Note that the optimal slack time does not depend on the cost rates (c_f, c_p, and c_t). Figure 3 shows the relationship between the percentage of optimal slack time (Δ^* / T_0) and paratransit demand under various ratios of unit deviation time to direct running time. The slack time should be increased as paratransit demand, dwell time at deviated stops, and the length of the route segment increase, and decreased as transit rider deviation tolerance decreases. When transit rider tolerance is high and paratransit demand is low, the optimal slack time mainly depends on paratransit demand and unit time per deviation.

Optimal Slack Time Distribution

Perhaps the most likely use of the analytical formula is in determining the optimal ratios by which to distribute a given amount of route slack time to individual route segments. In situations in which paratransit demand is relatively low, the optimal slack time should be dominated by the first item in Equation 13; therefore, the total slack time should be distributed in proportion to the product of paratransit demand (N_p) and zone size (δ). In the case that all route segments have similar zone size, the distribution ratio should be proportional to paratransit demand. This finding is consistent with the empirical results of Durvasula et al. (*3*).

However, when both transit and paratransit demand are high, the optimal slack time should be proportional to the direct running time; that is, the higher the direct running time between a segment is, the larger slack time should be allocated.

Effect of Zone Size on Feasible Deviations

As shown in Equation 5, the size of the service zone (w) is an important factor influencing the number of feasible deviations that can be accommodated by flex-route vehicles under a given amount of slack time. The larger a service zone is, the larger the average distance from deviated stops to the main route, and the more time is needed to visit a deviated stop, therefore, the fewer number of stops that can be accommodated. This conclusion has also been shown empirically by Durvasula et al. (*3*). It should, however, be pointed out that this effect of zone size is mainly due to the assumed service policy of the first-come-first-served policy. This operating policy is necessary in order to guarantee equitable access to the service, especially when the deviation service is also made available to the general public. However, it may not be necessary when the deviation service is available only to paratransit users such as the elderly and disabled. Rejecting these trips would not cause any judicial problems because they will be covered by a specialized paratransit service anyway. Therefore, if only paratransit trips are to be served and they are known in advance to the flex-route operator, there will be no advantage to restrict service to a given buffer area. By designating a larger service area, the operator will have more choices of deviations and therefore have a higher likelihood of (*a*) maximizing the number of deviations that can be accommodated and (*b*) minimizing the possible leftover slack time or idle time at the fixed stops.

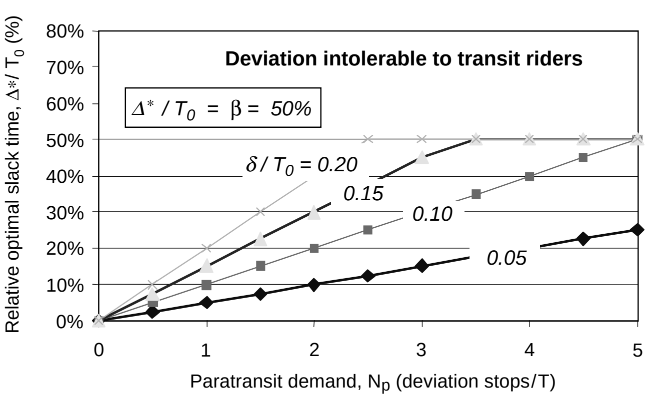

FIGURE 3 Optimal slack time as a function of paratransit demand and unit deviation travel time.

Effect of Dwell Time on Feasible Deviations

Equation 5 also reveals the effect of dwell times at deviated stops on the expected number of feasible stops that can be made within a given amount of slack time. As would be expected, the number of feasible deviations is inversely proportional to the average dwell time at each deviated stop. This relationship is consistent with the empirical analysis of Durvasula et al. (*3*). A further examination of the equation indicates that the relative effect depends on the associated service zone size (w) and the average vehicle running speed (v). This pattern can be shown using a simple numerical example in which a fixed vehicle running speed of 20 km/h is assumed. If the service zone size is 1200-m wide on each side, a dwell time of 60 s would yield $n_p = 10 / [(1.2 * 60 / 20) + 1.0] = 2.17$ feasible deviations. If the dwell time was increased to 66 s (a 10% increase), the average number of feasible deviations would be reduced to $10 / [(1.2 * 60 / 20) + 1.1] = 2.13$, which is a 3% reduction. However, if the zone size were instead 400 m, the same amount of increase in dwell time would induce a 9% of reduction in total number of feasible deviations.

SIMULATION ANALYSIS

The analytical model discussed in the previous section was established on the basis of several important assumptions including idealized network, continuous approximation of route length, and deterministic demand. The model has the following limitations:

1. It may overestimate the number of feasible deviations that can be made within a given amount of slack time.
2. It cannot predict the idle time at the end stop or intermediate fixed stops, which is clearly important to the transit riders who have to wait for the bus to depart during the idle time.
3. It identifies optimal slack time for a single route segment. However, it is unclear how this result can be applied to determine the optimal slack time for a flex-route system with multiple segments.

The objective of this section is to address these limitations through a simulation analysis. The simulation is performed using a simulation model called SimParatransit, which was originally developed as a tool for evaluating paratransit systems under a variety of operating conditions and service concepts (*5*). The system was extended with the functionality to model flex-route service. The simulation experiments were performed on a hypothetical network as shown in Figure 4. The following specifications were used:

1. The service area covers a corridor of two subzones labeled Zone 1 (5.6 km × 2.4 km) and Zone 2 (1.6 km × 1.2 km). Each zone is covered by a uniform grid road network with all neighboring nodes (intersections) connected by two links, one in each direction. Each link has a length of 400 m and a speed 20 km/h.
2. All service vehicles are identical and each has a seating capacity of 30 passengers. (Assuming this large capacity is to eliminate the possibility of rejecting trips because of the capacity constraint.) Vehicles run sequentially from Stops A to B and then to Stop C with a fixed headway of 30 min. Departure time at each fixed stop was created on the basis of the direct running times between the fixed stops and a prespecified slack time. For example, if the slack time is 14-min total, of which 10 min are allocated to Zone 1 and 4 min to Zone 2, and the scheduled departure time at Stop A is 8:00 a.m., the departure times at Stops B and C would be 8:27 a.m. and 8:36 a.m., respectively.
3. Two groups of paratransit trips are modeled. The first group of trips originates in Zone 1 and arrives at the same location—Stop B, whereas the second group of trips is from Zone 2 to the same location—Stop C. Trip origins of both groups are uniformly distributed over their corresponding originating zone and represent the deviation demand for the flex-route service. Each trip is assumed to have a pickup dwell time of 2 min and a drop-off dwell time of zero. The simulated demand rates are 2.5 trips/h for both groups, which were used to generate deviation requests with desired pickup times based on a stationary Poisson distribution.

For each simulation experiment setting, the service system was simulated continuously for 20 h, including a total of 40 bus trips from Stop A to Stop C, which should provide statistically reliable estimates of various performance measures. The following section presents key findings obtained from the simulation experiments.

Effects of Slack Time on Feasible Deviations

The first set of experiments focuses on the relationship among the number of deviated stops accommodated by the flex-route vehicles, the idle time at a fixed stop, and the assigned slack time. Only the route segment in Zone 1 (A-B) was allocated with slack time with values varying from 0 to 15 min. The simulated and theoretical results are shown in Figure 5 for the relationship between the percentage of feasible deviations and allocated slack time (Zone 1 only). It can be observed that the theoretical approximation (Equation 5 with $\delta = w/v + \tau = (60 * 1.2/20) + 2 = 5.6$ min) has captured the general trend of the relationship between the number of feasible deviations and the amount of slack time. However, the theoretical formula has consistently overestimated the number of feasible deviations. The overestimation is mainly caused by the variation in paratransit demand. Under a small amount of slack time (although on average the demand is higher than what can be accommodated) the variation in demand will cause some unsaturated periods, which would then lead to leftover slack time. When the slack time is increased to a certain point, the deviation demand becomes too low to use up all of the assigned slack time—that is why the overestimation tends to increase as the slack time increases.

Effect of Slack Time on Idle Time Distribution

Based on the analytical model, the idle time at a fixed stop should be equal to $\min\{0, \Delta - N_p\delta\}$, which would then predict zero idle time when the average deviation demand is high and the allocated slack time is small, or, $\Delta < N_p\delta$. However, because of the spatial and temporal variation in deviation demand, there may not be a sufficient number of requests in some periods to completely fill in the allocated slack time, even when the average demand is much higher than what the flex-route system can accommodate with the given slack time. This means a flex-route vehicle may have to wait (idle) at a fixed stop for its scheduled departure. The resulting idle time has a significant design implication as transit users have to sit and wait for the bus to depart and are likely to develop negative perceptions and attitudes toward the provided service. Figure 6 shows results from simulation and an analytical model under a given level of demand (note that in the analytical model, idle time = $\min\{0, \Delta - N_p\delta\}, = \min\{0, \Delta - 2 * 5.6\}$). As expected, the analytical model significantly underestimates the idle time. It can also be observed that the mean and standard deviation of the idle time at Stop B were approximately in proportion to the assigned slack time. This suggests that although use of larger slack time will have the benefit of being able to cover larger number of deviations, it will have a negative consequence of larger idle time at fixed stops. This result is not revealed in a deterministic model but is important in designing a

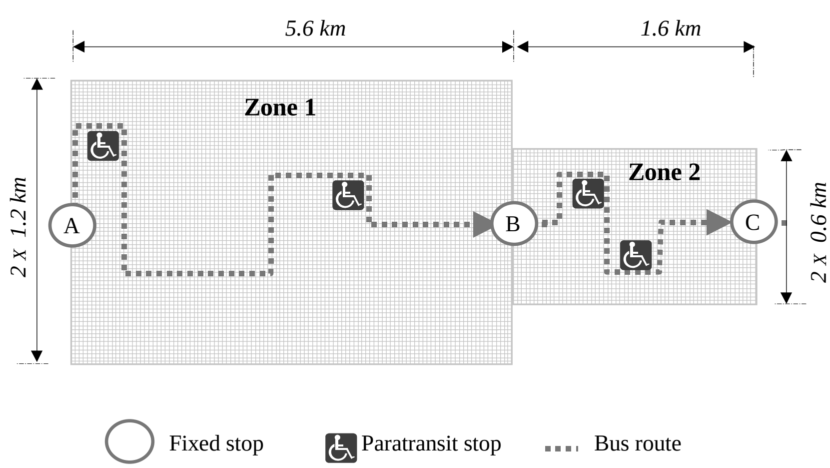

FIGURE 4 Flex-route service in an idealized network.

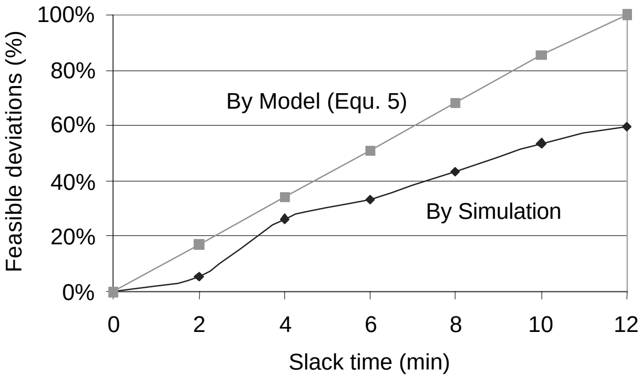

FIGURE 5 Feasible number of deviations versus slack time.

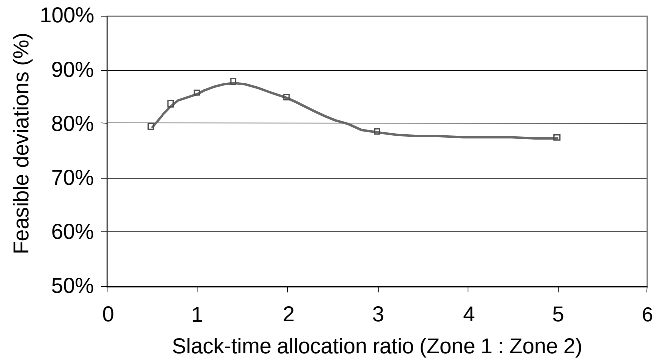

FIGURE 7 Feasible deviations versus slack-time distribution ratio.

flex-route system; this will be further examined when the issue of slack time allocation is discussed in the next section.

Slack Time Allocation for Maximal Number of Feasible Deviations

Figure 7 gives the relationship between the proportion of paratransit stops made by the flex-route service and the ratio of slack times allocated to Zone 1 and Zone 2. It is evident that there is an optimal allocation ratio at which the total number of paratransit trips covered by the flex-route service is maximized. In this simulated case, the optimal slack-time-allocation ratio is approximately 1.4. Note that this ratio is quite different from the distance-based allocation logic (2.0) and demand-based allocation method (1.0). Interestingly, it is close to what we would obtain based on the product of N_p and δ, or, $\delta_{zone1} / \delta_{zone2} = [(60 * 1.2 / 20) + 2] / [(60 * 0.6 / 20) + 2] = 1.47$. This evidence further supports the need to consider both the paratransit demand and demand area even when the regular transit demand is uniformly distributed over the route segments.

Figure 8 shows the mean and standard deviation of the total idle time at the fixed stops as a function of the slack-time-allocation ratio. Three important observations can be made. First, similar to the single zone case, the variation in idle time is fairly high no matter what ratio was used in allocating the total slack time. The coefficients of variation (which equal standard deviation and mean) range from 80% to 90%. Secondly, both mean and standard deviation of the idle time are fairly uniform across the slack-time-allocation ratio, suggesting that it may not be effective to reduce the idle time by selecting an appropriate slack-time-allocation ratio. With the results from the single zone simulation, it can be concluded that allocating a smaller amount of slack time is the only way to reduce idle time and idle time variation. As discussed previously, when a bus is idling with riders from previous segments on board, it will cause a negative view of the service and thus have a negative effect on future demand; this negative effect can be reduced by minimizing the number of fixed stops. An ideal system would be one with a single fixed stop (feeder service) or two fixed stops (flexible route shuttle).

CONCLUSIONS

Flex-route service represents an innovative integration of conventional fixed-route, fixed-schedule transit, and dial-a-ride demand-responsive service. The necessity to meet a fixed schedule and at the same time provide deviation service to other stops poses a significant challenge for the planning, design, and management of such services. This research has mainly focused on various issues associated with flex-route transit and is the first to approach the problem through a combination of theoretical and simulation analyses. Our proposed analytical model is simple but elaborate enough to reveal the fundamental relationships between system performance and design parameters.

The optimal slack time should be determined with a consideration of the trade-off between the savings that can be achieved from serving paratransit riders and the inconvenience that may result to the transit users. The critical factors that should be considered include level of paratransit demand, zone size, and paratransit dwell time. The

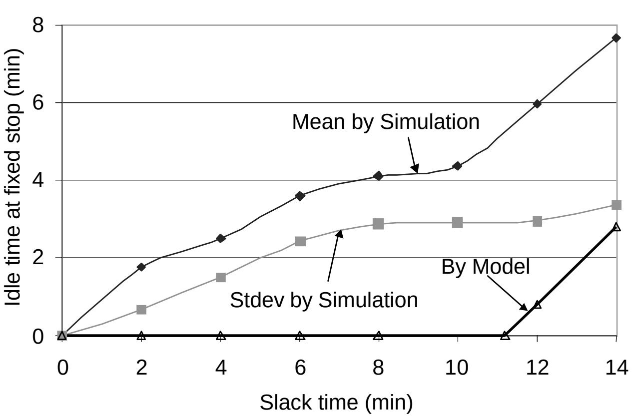

FIGURE 6 Idle time at fixed stop versus slack time (Stdev = standard deviation).

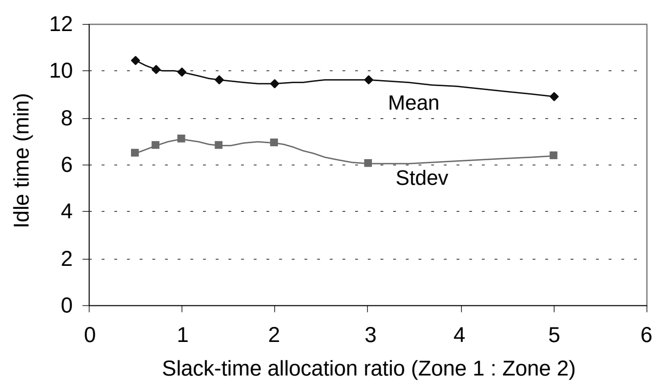

FIGURE 8 Idle time versus slack-time distribution ratio (Stdev = standard deviation).

proposed analytical model provides a framework for a systematic trade-off analysis in determining the optimal slack time as well as other design parameters such as zone size and length.

If the problem is just to distribute a given amount of slack time to individual route segments, the distribution scheme should consider paratransit demand and zone size. If the distribution objective is to maximize the total number of feasible deviations, the optimal distribution method should be based on the product of the expected paratransit demand and the average additional time that is required to visit a deviated stop.

Idle time at a fixed stop in the middle of a flex-route system has a negative effect on those riders who are already on the bus and are heading to a destination beyond the fixed stop. Unfortunately, idle time cannot be eliminated completely and in fact will increase as the allocated slack time increases. This finding suggests that the flex-route concept might be viable only with a minimal number of fixed stops, such as in feeder routes (one fixed stop) and shuttle routes (two fixed stops).

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REFERENCES

1. Rosenbloom, S. *Service Routes, Route Deviation, and General Public Paratransit in Urban, Suburban, and Rural Transit Systems.* AZ-26-7000. U.S. Department of Transportation, FTA, Washington, D.C., 1996.
2. Farwell, R. G., and E. Marx. Planning, Implementation, and Evaluation of OmniRide Demand-Driven Transit Operations: Feeder and Flex-Route Services. In *Transportation Research Record 1557,* TRB, National Research Council, Washington, DC, 1996, pp. 1–9.
3. Durvasula, K. P., B. L. Smith, S. C. Brich, and M. J. Demetsky. An Investigation of Route Deviation Transit Service Design. Presented at the 78th Annual Meeting of the Transportation Research Board, Washington, D.C, 1999.
4. Welch, W., R. Chisholm, D. Schumacher, and S. R. Mundle. Methodology for Evaluating Out-of-Direction Bus Route Segments. *Transportation Research Record 1308,* TRB, National Research Council, Washington, D.C., 1991, pp. 43–50.
5. Fu, L. Simulation Model for Evaluating Intelligent Paratransit Systems. In *Transportation Research Record: Journal of the Transportation Research Board, No. 1760,* TRB, National Research Council, Washington, D.C, 2001, pp. 93–99.

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